Solutions

Semester 1 Review – Part 1 **you should be able to solve all of these WITHOUT a calculator.**

$f(x)=ax^2+bx+c \Rightarrow$ axis of symmetry $x=-\dfrac{b}{2a}$ Discriminant: $\Delta=b^2-4ac$

The Quadratic Formula: $if\ ax^2+bx+c=0\ then\ x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Solve problems (1) and (2) by factorization:

1) $x^2-5x+6=0$

$(x-3)(x-2)=0$

$\rightarrow$ $\boxed{x=3 \text{ or } x=2 \text{ are sol'ns}}$

2) $2x^2-9x=-10 \rightarrow 2x^2-9x+10=0$

want $\rightarrow (2x-\ \ )(x-\ \ )=0$

need 2 #s that mult to +10 and make this work. Try some factors of 10. Both are Negative.

$(2x-5)(x-2)=0 \rightarrow$ $x=2$ or $x=5/2$

2) Solve the equation $3x^2-10x+2=0$ by using the quadratic formula

$a=3$

$b=-10$

$c=2$

$x=\dfrac{-(-10)\pm\sqrt{(-10)^2-4(3)(2)}}{2(3)}$

$x=\dfrac{10\pm\sqrt{100-24}}{6}$

$x=\dfrac{10\pm\sqrt{76}}{6}=\dfrac{10\pm\sqrt{4\cdot 19}}{6}$

$x=\dfrac{10\pm2\sqrt{19}}{6}$

$x=\dfrac{5\pm\sqrt{19}}{3}$

3) Determine how many Real roots (Zeros) the following quadratics have (without GRAPHING!)

a) $f(x)=3x^2-5x+10$ b) $g(x)=2x^2+3x-10$

use the Discriminant. IT tells us how many zeros there are

$\Delta>0 \rightarrow 2$ zeros

$\Delta=0 \rightarrow 1$ zero

$\Delta<0 \rightarrow$ No zeros

$a=3, b=-5, c=10$

$\Delta=(-5)^2-4(3)(10)$

$\Delta=25-120=-95<0$

$\rightarrow$ No zeros

$a=2, b=3, c=-10$

$\Delta=(3)^2-4(2)(-10)$

$=9+80=89>0$

$\rightarrow$ 2 zeros

4) Consider $f(x)=-kx^2+6x-1$, for $k\neq0$. The equation $f(x)=0$ has two equal roots. $\rightarrow \Delta=0$ (Discriminant)

(a) Find the value of k. $\Delta=(6)^2-4(-k)(-1)$

$\Delta=36-4k \rightarrow \Delta=0 \rightarrow 36-4k=0$

$4k=36 \rightarrow$ $\boxed{k=9}$

(b) The line $y=p$ intersects the graph of f. Find all possible values of p.

$\rightarrow$ $\boxed{p\leq0}$ for $y=p$ to intersect $f(x)$

$f(x)=-9x^2+6x-1$

has vertex on x-axis ($\Leftarrow$ 2 equal Roots) & is Reflected $\rightarrow$

5) Express $f(x)=x^2+6x-14$ in the form $f(x)=(x-h)^2+k$, where h and k are to be determined. In other words, figure out what h and k must be. Does this function have a maximum or minimum?

Find coord. of vertex!

axis of symmetry $=-\dfrac{6}{2(1)}=-3$

$\rightarrow$ Vertex $=(-3, f(-3))$

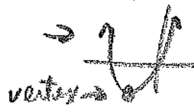
$f(-3)=(-3)^2+6(-3)-14=9-18-14=-23$

Vertex $=$ $\boxed{(-3,-23)=(h,k)}$

$\Rightarrow f(x)=(x--3)^2-23=(x+3)^2-23$

we do not have a Reflection

$\rightarrow$

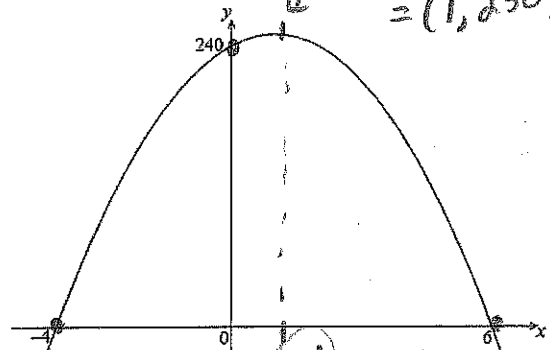


$\rightarrow$ Vertex is Minimum

6) The following diagram shows part of the graph of a quadratic function f.

$V=(h,k)$
$=(1,250)$

The x-intercepts are at $(-4, 0)$ and $(6, 0)$ and the y-intercept is at $(0, 240)$.



(a) Write down $f(x)$ in the form $f(x) = -10(x-p)(x-q)$.

$p=-4$
$q=+6$ $\rightarrow \boxed{f(x)=-10(x+4)(x-6)}$

(b) Find another expression for $f(x)$ in the form $f(x) = -10(x-h)^2 + k$. vertex $\frac{1}{2}$ way between zeros

$h=1$ $h=\frac{6+-4}{2}=\frac{2}{2}=1$

$V=(1,f(1))$ $V=(1,250)$

$f(1)=-10(1+4)(1-6)=-10(5)(-5)=250 \longrightarrow \boxed{f(x)=-10(x-1)^2+250}$

(b) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$.

Expand $f(x)$ from part (a) $f(x)=-10(x+4)(x-6) = -10[x^2-6x+4x-24]$
$=-10[x^2-2x-24]$
$\boxed{f(x)=-10x^2+20x+240}$

(c) A particle moves along a straight line so that its velocity, v in meters per second, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \le t \le 6$. $V(t)$ is same as $f(x)$

(i) Find the value of t when the speed of the particle is greatest.

V is greatest at its Max. from part (b) max at $t=1$, $V(1)=250 \rightarrow \boxed{t=1 \text{ sec.}}$

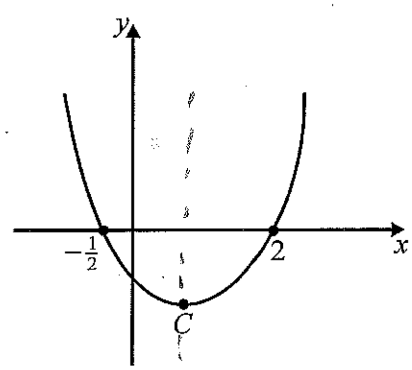
(ii) What is the particle's initial velocity?

Initial velocity is at $t=0$
$\Rightarrow$ = y-intercept

$\boxed{V(0)=240 \text{ m/sec}}$

7) The diagram represents the graph of the function

$$f : x \mapsto (x-p)(x-q).$$



(a) Write down the values of p and q.

$p=-\frac{1}{2}$
$q=2$ } they are the Zeros!
$\rightarrow f(x)=(x+\frac{1}{2})(x-2)$

(b) The function has a minimum value at the point C. Find the coordinates of C.

C is Vertex h = Middle of the zeros. $f(3/4)=(\frac{3}{4}+\frac{1}{2})(\frac{3}{4}-2)$

$C=(h,k)$ $h=\frac{-\frac{1}{2}+2}{2}=\frac{3/2}{2}=3/4$ $=(5/4)(-5/4) = -25/16$

$\boxed{C=(\frac{3}{4}, -\frac{25}{16})}$

(c) Describe the transformation that changes the function $g(x) = x^2$ into $f(x)$ above.

$f(x)=(x-\frac{3}{4})^2-\frac{25}{16}$

$\rightarrow$ $\boxed{\text{Translate Right } \frac{3}{4} \text{ and Translate down } \frac{25}{16}}$