Solutions

SL Mathematics Year 1 Sem1_Final_Review2019.docx

Semester 1 Review – Part 4: Logarithms. You may use calculators on these problems

Exponential and Logarithm Properties:

$$a^x = b \iff x = \log_a b \qquad \log_c a + \log_c b = \log_c ab \qquad \log_c a - \log_c b = \log_c \frac{a}{b} \qquad \log_c a^r = r\log_c a \qquad \log_b a = \frac{\log_c a}{\log_c b}$$

1) Write each of the following using a single logarithmic expression, or, if possible, without any logarithms.

a. $\log(5) - 4\log(x) =$

$= \log(5) - \log(x^4)$

$\boxed{= \log\left[\frac{5}{x^4}\right]}$

b. $\log_2(16^a) = \log_2((2^4)^a) = \log_2(2^{4a})$

$= 4a\log_2(2) \boxed{= 4a}$

2) Given than $X = \log_a(2)$, $Y = \log_a(3)$ *and* $Z = \log_a(5)$ write the expressions below in terms of X, Y and Z. Your results should not have any logarithms in them.

a. $\log_a(15) = \log_a(3\cdot 5)$

$= \log_a(3) + \log_a(5)$

$\boxed{= Y + Z}$

b. $\log_a\left(\frac{4}{\sqrt{3}}\right) = \log_a(4) - \log_a(\sqrt{3})$

$= \log_a(2^2) - \log_a(3^{1/2})$

$= 2\log_a(2) - \frac{1}{2}\log_a(3) = \boxed{2X - \frac{1}{2}Y}$

3) Solve for the value of x in the equation: $\log_4(3x + 5) = 2$

$\log_4(3x+5) = 2 \iff 3x+5 = 4^2$

$3x+5 = 16$

$3x = 11$

$\boxed{x = 11/3}$

4) Solve the equation $\log_2(x) + \log_2(x-2) = 3, for\ x > 2$.

$\log_2(x) + \log_2(x-2) = 3$

$\log_2[x(x-2)] = 3$

$\iff x(x-2) = 2^3 = 8$

quadratic

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

But $x = -2$ is not possible

$\Rightarrow \boxed{x = 4}$

5) a) Given the function $f(x) = \log_3(\sqrt{x})$, show that $f^{-1}(x) = 3^{2x}$.

$y = \log_3(\sqrt{x}) \rightarrow x = \log_3(\sqrt{y}) \longleftrightarrow 3^x = \sqrt{y} \rightarrow (3^x)^2 = (\sqrt{y})^2 = y$

$y = (3^x)^2 \rightarrow y = \boxed{3^{2x} = f^{-1}(x)}$ ✓

b) What is the Range of $f^{-1}(x)$?

Range of $f^{-1}(x)$ = Domain of $f(x)$

Dom $f(x) = \{x \mid x > 0\} \rightarrow \boxed{\text{Range } f^{-1}(x): \{y \mid y > 0\}}$

c) Given that $g(x) = \log_3(x)$, find $(f^{-1} \circ g)(2)$. Write your answer as an integer.

$(f^{-1}\circ g)(2) = f^{-1}(g(2)) = f^{-1}(\log_3(2)) = 3^{2(\log_3(2))} = 3^{\log_3(2^2)} = 2^2 \boxed{= 4}$

Add exponents

6) Simplify each of the following to a Rational Number or Solve for x.

a) $\log_x(x^2 \cdot \sqrt[5]{x}) = \log_x(x^2 \cdot x^{1/5})$

$= \log_x(x^{2\frac{1}{5}}) = \boxed{2\tfrac{1}{5}}$

b) $\log_2(x) = 3 \iff x = 2^3 = 8$

$\boxed{x = 8}$

7) Given that $(2^x)^2 + (2^x) - 12$ can be written as $(2^x + a)(2^x + b)$ where a and b are integers, find the value of a and of b. This is like factoring a Quadratic. Need $a \cdot b = -12$, $a + b = +1$

$(2^x)^2 + (2^x) - 12 = (2^x + 4)(2^x - 3)$

$\Rightarrow \boxed{a = 4,\ b = -3}$

Hence find the exact solution of the equation $(2^x)^2 + (2^x) - 12 = 0$, and explain why there is only one solution.

$\Rightarrow 2^x + 4 = 0$ or $2^x - 3 = 0$

$2^x = -4$ $\quad$ $2^x = 3$ (apply $\log_2(\cdot)$ to both sides)

No sol'ns! $\quad$ $\rightarrow x = \log_2(3)$

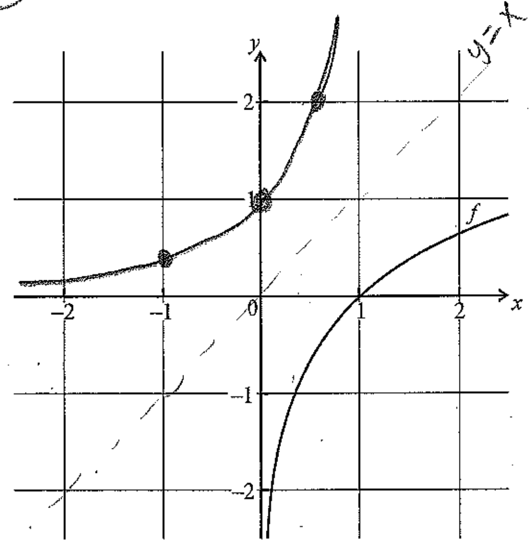
8) Let $f(x) = \log_a(x)$

a. Write down the value of:

i. $f(a) =$ 1 $\quad \log_a(a) = 1$ always!

ii. $f(1) =$ 0 $\quad \log_a(1) = 0$ always $(a^0 = 1)$

iii. $f(a^4)$ 4 $\quad \log_a(a^4) = 4 \cdot \log_a(a) = 4 \cdot 1$



b. The diagram on the right shows part of the graph of f.

On the same diagram, sketch the graph of f^{-1}.

Reflect curve over line $y = x$

$[f^{-1}(x) = a^x]$

9) Use the function $f(x) = \ln(x + 2) - 1$ to answer the questions below.

a. Determine the Domain and the Asymptote for this function.

$x + 2 > 0$

$\rightarrow x > -2$ $\quad$ Domain: $\{x \mid x > -2\}$

Vertical

Asymptote: $\boxed{x = -2}$

b. Determine the inverse function, $f^{-1}(x)$, for this function.

$x = \ln(y + 2) - 1$

$\Rightarrow x + 1 = \ln(y + 2) = \log_e(y + 2)$

$\Rightarrow e^{x+1} = y + 2 \rightarrow y = e^{x+1} - 2$

$\boxed{f^{-1}(x) = e^{x+1} - 2}$

c. Determine $f^{-1}(3) + f^{-1}(-1) =$ 51.598

$f^{-1}(3) = e^{3+1} - 2 = e^4 - 2$

$f^{-1}(-1) = e^{-1+1} - 2 = e^0 - 2 = 1 - 2 = -1$

$f^{-1}(3) - f^{-1}(1) = e^4 - 2 + (-1)$

$\boxed{= e^4 - 3 = 51.598}$