Semester 1 Review – Part 3: Exponential Functions. You may $\boxed{\text{use calculators}}$ on these problems

1. A population of bacteria is growing at the rate of 15% per hour. There are currently 500 spores of this bacteria in the sample.

Exponential growth $Y = a\cdot(1+r)^t$

a) Create an equation that gives the population for this bacteria at any time t (t is in hours).

$r = 15\% \rightarrow r = 0.15$ as a decimal

$P(t) = 500(1+.15)^t \boxed{= 500(1.15)^t}$

b) How any bacteria will there be in 10 hours?

$P(10) = 500(1.15)^{10}$

≈ 2023 spores

c) How long will it take for the size of the population to double? Give your answer to the nearest hour.

$500(1.15)^t = 1000$

$\rightarrow (1.15)^t = \frac{1000}{500} = 2$

use calculator (graph or table) or use logs.

$(1.15)^t = 2 \rightarrow \log[(1.15)^t] = \log(2)$

$t\cdot\log(1.15) = \log(2) \rightarrow t = \frac{\log(2)}{\log(1.15)} \boxed{= 4.96\text{ hr}}$ $\boxed{t \approx 5\text{ hr}}$

2. Initially a tank contains 10,000 litres of liquid. At the time $t = 0$ minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, V litres, which remains in the tank after t minutes is given by:

$$V = 10000 \cdot (0.933)^t$$

Exponential Decay

(a) What percent of the liquid is flowing out each minute?

$.933$ is portion remaining $\Rightarrow 1 - .933 = 0.067$ = portion flowing out $\rightarrow$ $\boxed{6.7\%\text{ liquid flows out each minute}}$

(b) Find the value of V after 5 minutes.

$V(5) = 10000(.933)^5$

$\boxed{= 7070\text{ litres}}$

(c) Find how long, to the nearest tenth of a minute, it takes for half of the initial amount of liquid to flow out of the tank. want $V = 5000$ ($\frac{1}{2}$ Remain) Use logs

$10000(.933)^t = 5000$

$\rightarrow (.933)^t = \frac{5000}{10000} = \frac{1}{2}$

$(.933)^t = \frac{1}{2}$

$\log(.933)^t = \log(.5)$

$t\log(.933) = \log(.5)$

$t = \frac{\log(.5)}{\log(.933)} = 9.99$

$\boxed{t \approx 10\text{ minutes}}$

(d) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen.

want $V = .05 \times 10000 = 500$ litres

(a) $10000(.933)^t = 500 \rightarrow (.933)^t = .05 \rightarrow t\log(.933) = \log(.05)$ $t = \frac{\log(.05)}{\log(.933)} = 43.2$ min

(b) Put Formula into calc $\rightarrow$ Go to table & see that $t = 43 \rightarrow V \approx 500$

$t = 43\text{ min} \approx \frac{3}{4}$ hour

3. Rewrite in simplest rational form. Your final answer should not have any exponents.

Remember $\sqrt[n]{x} = x^{1/n}$

a. $\sqrt[3]{125^2} = \sqrt[3]{5^6}$

$= \sqrt[3]{(5^3)^2} = \sqrt[3]{5^6}$

$= (5^6)^{1/3} = 5^{6/3} = 5^2 \boxed{= 25}$

b. $32^{-4/5} = (2^5)^{-4/5} = 2^{5\cdot(-\frac{4}{5})}$

$= 2^{-4} = \frac{1}{2^4}$

$\boxed{= \frac{1}{16}}$

4. Write in the form a^x where a is a prime number and x is a rational number.

$$16 \cdot 2^{\frac{3}{5}} = 2^4 \cdot 2^{3/5} = 2^{4+3/5} = 2^{\frac{20}{5}+\frac{3}{5}} \boxed{= 2^{23/5}} \text{ or } 2^{4\frac{3}{5}}$$

(Add exponents)

5. Ella just put \$500 into a bank account that promises to pay 3.0% annual interest, compounded monthly.

a) Create a formula that will give the amount of money, M(t), that Ella will have for any month (t). Make sure you reflect the monthly compounding.

Monthly rate $= \frac{3\%}{12} = .25\%$

$\rightarrow$ r = .0025

$$M(t) = 500(1+.0025)^t = \boxed{500(1.0025)^t}$$

b) How much money will Ella have saved up in three years if she leaves her money in this bank account?

36

$$M(36) = 500(1.0025)^{36} = \$547.03$$

3 yr = 3·12 mo = 36 month

c) When will she have enough to buy the new iPhone that costs \$1,100?

$$500(1.0025)^t = 1100$$

$$(1.0025)^t = \frac{1100}{500} = 2.2$$

$$(1.0025)^t = 2.2$$

use logs

$$t \log(1.0025) = \log(2.2)$$

$$t = \frac{\log(2.2)}{\log(1.0025)} = 315.8 \text{ months}$$

$316/12 = 26$ yrs!!

(yikes)

6. Use the functions $f(x) = 3x - 5$ and $g(x) = 4 \cdot e^x$ to answer the questions below:

a) Describe the asymptotes of the function g(x).

g(x) is Exponential $\rightarrow$ has horizontal asymptote (& no vertical shift)

$\rightarrow$ $\boxed{y=0}$ is Horiz. Asymptote

b) Determine the composite function $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x)) = f(4 \cdot e^x) = 3(4 \cdot e^x) - 5$$

$$\boxed{(f \circ g)(x) = 12e^x - 5}$$

c) Calculate $(f \circ g)(-2)$

$$(f \circ g)(-2) = 12 \cdot e^{(-2)} - 5 \approx -3.38$$

d) Describe the asymptotes of the function $(f \circ g)(x)$.

$(f \circ g)(x) = 12e^x - 5$ is also exponential

AND it is a Transformation of g(x) by a vert stretch factor = 3 then vertical shift down 5.

$\rightarrow$ Horizontal asymptote is stretched up by factor of 3 (but $3 \cdot 0 = 0$ $\rightarrow$ No change)

then shifted down 5 $\rightarrow$ New Horizontal Asymptote $\Rightarrow$ $\boxed{y=-5}$