Solutions

SL Mathematics Year 1 Sem1_Final_Review2019.docx

Semester 1 Review – Part 2: Functions

1. Consider the function $f: x \mapsto \sqrt{x+1},\ x \geq -1$ $\rightarrow$ $y = \sqrt{x+1}$

(a) Determine the inverse function f^{-1}.

$f^{-1}(x)$: $x = \sqrt{y+1}$

(flip x & y and solve for y) $x^2 = y+1$

$\boxed{y = x^2 - 1}$ or $\boxed{f^{-1}(x) = x^2 - 1}$

(b) What is the domain of f^{-1}?

Domain of $f^{-1}(x)$ is the same as Range of $f(x)$

Range of $\sqrt{x+1}$ is $\{y \mid y \geq 0\}$ $\longrightarrow$ $\boxed{\text{Domain } f^{-1}(x) \text{ is } \{x \mid x \geq 0\}}$

2. Use the functions $f(x) = 2x - 3$ and $g(x) = x^2 + x$ to answer the questions below:

a) Determine the composite function $(f \circ g)(x)$

$(f \circ g)(x) = 2 \cdot g(x) - 3 = 2[x^2 + x] - 3$ $\boxed{= 2x^2 + 2x - 3}$

means Apply g(x) first, then f(x)

b) Determine the composite function $(g \circ f)(x)$. Does this equal $(f \circ g)(x)$?

$(g \circ f)(x) = g(f(x)) = (f(x))^2 + f(x) = (2x-3)^2 + (2x-3) = (2x-3)(2x-3) + (2x-3)$

$= 4x^2 - 12x + 9 + 2x - 3$

$(g \circ f)(x) = 4x^2 - 10x + 6$ $\rightarrow$ $(g \circ f)(x) \neq (f \circ g)(x)$

order Matters!

c) Evaluate $(g \circ f)(3)$

$(g \circ f)(3) = 4(3)^2 - 10(3) + 6$

$= 36 - 30 + 6$

$\boxed{(g \circ f)(3) = 12}$

or we could do $(g \circ f)(3) = g(f(3))$

$f(3) = 2(3) - 3$

$f(3) = 3$ $\longrightarrow$ then $g(f(3)) = g(3) = 3^2 + 3 = 9 + 3$

$= 12$

3. Suppose $f(x) = ax + b$ where a and b are constants. If $f(1) = 7$ and $f(3) = -5$ then determine the values of a and b. Use a and b to calculate $f(10)$.

$f(1) = a \cdot 1 + b = 7$ $\Rightarrow$ $\boxed{\begin{array}{l} a + b = 7 \\ 3a + b = -5 \end{array}}$

$f(3) = a \cdot 3 + b = -5$

solve a system of 2 eqns

$a + b = 7$

$-[3a + b = -5]$

$a - 3a = 7 + 5 = 12$

$-2a = 12$

$\boxed{a = -6}$ $\rightarrow$ $-6 + b = 7$

$\boxed{b = 13}$

$\boxed{f(x) = -6x + 13}$

so, $f(10) = -6(10) + 13$

$= -60 + 13$

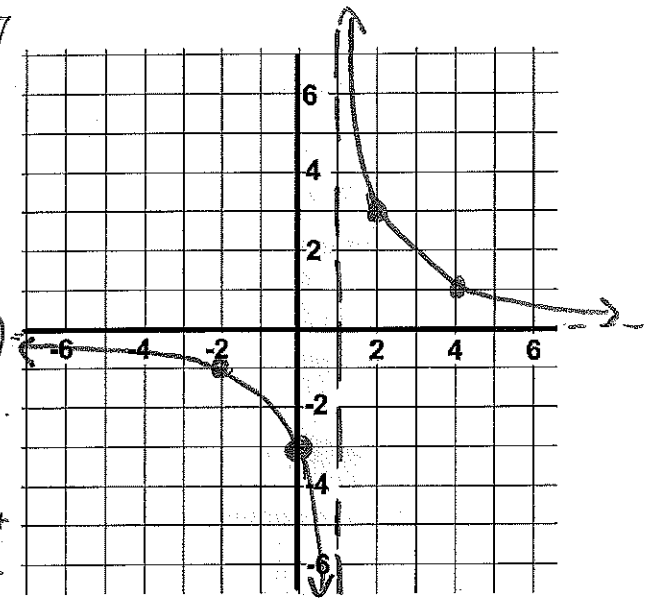
$\boxed{f(10) = -47}$

4. Use the function $f(x) = \frac{3}{x-1}$ to answer the following:

a) Find the axes intercepts for the function.

y-int : $x=0 \Rightarrow f(0) = \frac{3}{0-1} = \frac{3}{-1} = -3 \Rightarrow \boxed{(0,-3)}$

x-int $y=0 \rightarrow \frac{3}{x-1} = 0$ This can never happen! $\rightarrow$ No x-intercepts



b) Determine the Domain and Range for f(x).

Domain: $\{x \mid x \neq 1\}$

Range: $\{y \mid y \neq 0\}$

f(1) Doesn't exist (Divide by $\emptyset$)

c) Identify all the asymptotes of this function.

Vert. Asymptote : $x = 1$

Horiz Asymptote : $y = 0$

as $x \to \infty$, $y \to 0^+$

as $x \to -\infty$, $y \to 0^-$

d) Sketch a graph of the function at the right. find a few more pts $f(2) = \frac{3}{1} = 3$, $f(11) = 0.3$

e) Calculate $f^{-1}(x)$.

Swap x and y and solve for y:

$x = \frac{3}{y-1}$

$x(y-1) = 3$

$xy - x = 3$

$xy = 3 + x$

(now divide by x)

$\boxed{y = \frac{3+x}{x}}$ or $\boxed{f^{-1}(x) = \frac{3+x}{x}}$

f) Evaluate $(f^{-1} \circ f)(15)$.

$(f^{-1} \circ f)(x) = x$ for all values of x in Domain of $f(x)$

this is property of inverses $\Rightarrow$ $\boxed{(f^{-1} \circ f)(15) = 15}$

5. The function $f(x) = a(x-h)^2 + k$ is a quadratic function in Vertex Form. Use the following information to determine the constants $a, h,$ and k in this function.

The Axis of Symmetry is given by the equation: $x = 2$ **and we know** $f(2) = 4$ **and** $f(4) = 0$.

axis of symmetry at $x=2 \rightarrow h=2$ (x-coord of vertex)

$f(2)=4$ means $(2,4)$ is a pt on the graph but $x=2$ is where vertex is

$\Rightarrow k=4$ (y-coord of the vertex)

$\rightarrow f(x) = a(x-2)^2+4$ now use $f(4)=0$ to find a:

$a(4-2)^2+4=0 \rightarrow 4a+4=0 \rightarrow \boxed{a=-1}$

$\boxed{f(x) = -(x-2)^2+4}$

6. Simplify each of the following and **do not use** any negative exponents in your final answer.

a. $\frac{p^{18}}{p^{11}} =$

subtract Exponents

$= p^{18-11}$

$\boxed{= p^7}$

b. $(x^{-3}y)^{-4} =$

(mult exponents)

$= x^{(-3)(-4)} \cdot y^{1(-4)}$

$= x^{+12} y^{-4} = \boxed{\frac{x^{12}}{y^4}}$

c. $\frac{12x^2y^5 \cdot (xy)^0}{y^8} =$ $(xy)^0 = 1$

$\frac{12x^2y^5}{y^8}$

$= 12x^2y^{5-8} = 12x^2y^{-3}$

$\boxed{= \frac{12x^2}{y^3}}$